CHAPTER-1 [RELATION AND FUNCTION]

① $2 \in A \Rightarrow (2,2) \notin R$, $\therefore$ R is not reflexive.
$(2,3) \in R \Rightarrow (3,2) \notin R$, $\therefore$ R is not symmetric.
$(2,3) \in R$, $(3,1) \in R \Rightarrow (2,1) \notin R$ $\therefore$ R is not Transitive.

② $fog(x) = f\{g(x)\} = f(x^2+5) = 2(x^2+5)+3 = 2x^2+13$
$gof(x) = g\{f(x)\} = g(2x+3) = (2x+3)^2+5$

③ $fog(1) = f(g(1)) = f(3) = 1$
$fog(3) = f(g(3)) = f(3) = 1$
$fog(4) = f\{g(4)\} = f(9) = 3$
$fog(5) = f\{g(5)\} = f(9) = 3$
$\therefore fog = \{(1,1), (3,1), (4,3), (5,3)\}$.
Similarly $gof = \{(3,3), (9,3), (12,9)\}$.

④ $3 * 4 = 3\times 3 + 4 - 3 = 9 + 4 - 3 = 10$ A

⑤ $\because (a*b)*c = (ab+1)*c = (ab+1)c+1$
$= abc+c+1$
$a*(b*c) = a*(bc+1) = a(bc+1)+1 = abc+a+1$
$(a*b)*c \neq a*(b*c)$ $\therefore$ not associative.

⑥ $fof(x) = f\{f(x)\} = f\left(\frac{4x+3}{6x-4}\right)$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)}$$

$$= \frac{34x}{34}$$

$$= x$$

(7) $x+2y=8 \Rightarrow y = \frac{8-x}{2}$

Range $= \{3, 2\}$.

(8) Reflexivity: $\because \forall\, a \in A \Rightarrow |a-a|$ is even

$\Rightarrow (a,a) \in R$

$\therefore$ R is reflexive.

Symmetry: Let $(a,b) \in R \Rightarrow |a-b|$ is even

$\Rightarrow |b-a|$ is even

$\Rightarrow (b,a) \in R$

$\therefore$ R is symmetric.

Transitivity.

Let $(a,b) \in R$, $(b,c) \in R$

$\Rightarrow |a-b|$ is even, $|b-c|$ is even

$\Rightarrow |a-b+b-c|$ is even

$\Rightarrow |a-c|$ is even

$\Rightarrow (a,c) \in R$ $\therefore$ R is Transitive.

$\therefore$ R is an equivalence relation.

~~|4+1| is even, |1-3| is even~~

as we know that the difference of any two odd nos. is even and the difference of any two even no. is even, $\therefore$ all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other.

But the difference of an even no. and an odd no is not even (odd)

$\therefore$ no element of $\{1, 3, 5\}$ are related to any element of $\{2, 4\}$.

(9) $f(1) = \frac{1+1}{2} = 1$, $f(2) = \frac{2}{2} = 1$

$1 \neq 2 \Rightarrow f(1) = f(2)$

$\therefore f$ is many one.

Let n be any arbitrary element of N.
If n is odd, then $2n-1$ is also an odd natural number. such that

$$f(2n-1) = \frac{2n-1+1}{2} = n.$$

If n is even, then $2n$ is also even, s.t.

$$f(2n) = \frac{2n}{2} = n$$

Every element $n \in N$ (co-domain) has its pre-image in domain.

$\therefore f$ is many-one and onto.

(10) Let ~~A~~ R and ~~B~~ S are any two equivalences on a set A. Then $R \subseteq A \times A$ & $S \subseteq A \times A$

$\Rightarrow R \cap S \subseteq A \times A$

$\Rightarrow R \cap S$ is also a relation on A.

Reflexivity: Let $a \in A \Rightarrow (a,a) \in R$, $(a,a) \in S$

[$\because$ R & S are reflexive]

$\Rightarrow (a,a) \in R \cap S$

$\Rightarrow R \cap S$ is also reflexive.

Symmetry: Let $(a,b) \in R \cap S$

$\Rightarrow (a,b) \in R$ & $(a,b) \in S$ [by defn]

$\Rightarrow (b,a) \in R$ & $(b,a) \in S$ [R & S are symmetric]

$\Rightarrow (b,a) \in R \cap S$

$\Rightarrow R \cap S$ is also symmetric.

Transitivity:

Let $(a,b) \in R \cap S$, $(b,c) \in R \cap S$

$\Rightarrow (a,b) \in R$, $(a,b) \in S$, $(b,c) \in R$, $(b,c) \in S$

[by defn].

$\Rightarrow \{(a,b) \in R$ & $(b,c) \in R\}$ & $\{(a,b) \in S, (b,c) \in S\}$

$\Rightarrow$ $(a,c) \in R$ & $(a,c) \in S$

[R & S are Transitive]

$\Rightarrow (a,c) \in R \cap S$

$\therefore$ $R \cap S$ is Transitive.

$\therefore$ $R \cap S$ is also an equivalence relation

(11) Commutativity: $\because \forall A, B \in P(X) \Rightarrow A \cup B = B \cup A$

$\Rightarrow *$ is commutative.

Associativity: Let $A, B, C \in P(X)$

$A*(B*C) = A*(B \cup C) = A \cup (B \cup C)$

$= (A \cup B) \cup C$

$= (A*B) \cup C = (A*B)*C.$

$\therefore *$ is associative.

Existence of Identity: Let $A \in P(X)$. If E is the identity element, then

$A*E = A \Rightarrow A \cup E = A$

$\Rightarrow E = \phi$

$\therefore \phi$ is only the identity element.

Inverse. Let $A \in P(X)$. If B is the inverse of A, then

$A*B = E$

$\Rightarrow A \cup B = \phi$

$\Rightarrow A = B = \phi.$

$\therefore \phi$ is only the invertible element.

CHAPTER-2 (INVERSE TRIG. FUNCTION)

(1) $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$

$= \tan^{-1}\left\{2\cos\left(2\times\frac{\pi}{6}\right)\right\} = \tan^{-1}\left\{2\cos\frac{\pi}{3}\right\}$

$= \tan^{-1}\left(2\times\frac{1}{2}\right) = \tan^{-1}1 = \frac{\pi}{4}$ A

(2) $\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{6} - 2\times\frac{\pi}{4}$

$= \frac{\pi}{6} - \frac{\pi}{2} = -\frac{\pi}{3}$ A

(3) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) + \operatorname{cosec}^{-1}\frac{2}{\sqrt{3}}$

$= \tan^{-1}\sqrt{3} - \left[\pi - \sec^{-1}2\right] + \operatorname{cosec}^{-1}\frac{2}{\sqrt{3}}$

$= \frac{\pi}{3} - \pi + \frac{\pi}{3} + \frac{\pi}{3} = 0$ A

(4) $\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \tan^{-1}\tan\left(\pi - \frac{\pi}{4}\right) = \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$= \tan^{-1}\tan\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4}$ A

(5) $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right) = \tan^{-1}\tan\left(\frac{\pi}{4} - x\right) = \frac{\pi}{4} - x$ A

(6) $\tan^{-1}\sqrt{\frac{a-x}{a+x}}$, put $x = a\cos\theta$

$\Rightarrow \cos\theta = \frac{x}{a}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{x}{a}\right)$.

$\tan^{-1}\sqrt{\frac{a-a\cos\theta}{a+a\cos\theta}}$

$= \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}} = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$

$= \frac{1}{2}\cos^{-1}\left(\frac{x}{a}\right)$ A

(7) $\tan^{-1}\left(\dfrac{\sqrt{1+\cos x}-\sqrt{1-\cos x}}{\sqrt{1+\cos x}+\sqrt{1-\cos x}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2\cos^2\frac{x}{2}}-\sqrt{2\sin^2\frac{x}{2}}}{\sqrt{2\cos^2\frac{x}{2}}+\sqrt{2\sin^2\frac{x}{2}}}\right)$

$= \tan^{-1}\left(\dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$

$= \tan^{-1}\left(\dfrac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)$

$= \tan^{-1}\cdot\tan\left(\frac{\pi}{4}-\frac{x}{2}\right) = \frac{\pi}{4}-\frac{x}{2}$ Ans

(8) $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right]$

$\because \sin(\cot^{-1}x) = \sin\left(\sin^{-1}\dfrac{1}{\sqrt{1+x^2}}\right) = \dfrac{1}{\sqrt{1+x^2}}$

$\cos\left[\tan^{-1}\dfrac{1}{\sqrt{1+x^2}}\right] = \cos\left\{\cos^{-1}\sqrt{\dfrac{1+x^2}{2+x^2}}\right\}$

$= \sqrt{\dfrac{1+x^2}{2+x^2}}$ Ans

(9) $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}$

$= \tan^{-1}\left[\dfrac{\frac{1}{5}+\frac{1}{3}}{1-\frac{1}{5}\times\frac{1}{3}}\right]+\tan^{-1}\left[\dfrac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right]$

$= \tan^{-1}\frac{8}{14}+\tan^{-1}\frac{15}{55}$

$= \tan^{-1}\frac{4}{7}+\tan^{-1}\frac{3}{11}$

$= \tan^{-1}\left(\dfrac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\times\frac{3}{11}}\right) = \tan^{-1}\left(\dfrac{44+21}{77-12}\right) = \tan^{-1}\frac{65}{65} = \tan^{-1}1$

$= \frac{\pi}{4}$ Ans

(10) (i) L.H.S.

$$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)$$

$$=\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)+\pi+\tan^{-1}\left(\frac{c-a}{1+ca}\right).$$

$$\left[\because \cot^{-1}x=\pi+\tan^{-1}\frac{1}{x},\ x<0\right].$$

$$=\tan^{-1}a-\tan^{-1}b+\tan^{-1}b-\tan^{-1}c+\pi+\tan^{-1}c-\tan^{-1}a$$

$$=\pi \quad \text{R.H.S.}$$

(ii) $\because \cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$

$$\Rightarrow \cos^{-1}\left\{\frac{x}{a}\times\frac{y}{b}-\sqrt{1-\frac{x^2}{a^2}}\cdot\sqrt{1-\frac{y^2}{b^2}}\right\}=\alpha$$

$$\Rightarrow \frac{xy}{ab}-\frac{\sqrt{a^2-x^2}\sqrt{b^2-y^2}}{ab}=\cos\alpha$$

$$\Rightarrow \frac{xy}{ab}-\cos\alpha=\frac{\sqrt{a^2-x^2}\sqrt{b^2-y^2}}{ab}$$

sq. both sides.

$$\frac{x^2y^2}{a^2b^2}+\cos^2\alpha-\frac{2xy}{ab}\cos\alpha=\frac{(a^2-x^2)(b^2-y^2)}{a^2b^2}$$

$$\Rightarrow \cancel{\frac{x^2y^2}{a^2b^2}}+\cos^2\alpha-\frac{2xy}{ab}\cos\alpha=\frac{a^2b^2}{a^2b^2}-\frac{a^2y^2}{a^2b^2}-\frac{b^2x^2}{a^2b^2}+\cancel{\frac{x^2y^2}{a^2b^2}}$$

$$\Rightarrow \frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=1-\cos^2\alpha$$

$$\Rightarrow \frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha.$$

(11) (i) $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left\{\dfrac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2} \times \frac{x+1}{x+2}}\right\} = \frac{\pi}{4}$

$\Rightarrow \dfrac{(x-1)(x+2) + (x+1)(x-2)}{(x-2)(x+2) - (x-1)(x+1)} = \tan\frac{\pi}{4}$

$\Rightarrow \dfrac{x^2 \cancel{+} \cancel{x} - 2 + x^2 - \cancel{x} - 2}{x^2 - 4 - x^2 + 1} = 1$

$\Rightarrow 2x^2 - 4 = -3 \Rightarrow 2x^2 = 1$

$\Rightarrow \boxed{x = \pm\frac{1}{\sqrt{2}}}$

(ii) $2\tan^{-1}(\cos x) = \tan^{-1}(2\cos x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2\cos x)$

$\Rightarrow \frac{\cancel{2}\cos x}{\sin^2 x} = \cancel{2}\cos x$

$\Rightarrow \cos x - \cos x \sin^2 x = 0$

$\Rightarrow \cos x(1 - \sin^2 x) = 0.$

$\Rightarrow \cos^3 x = 0$

$\Rightarrow \cos x = 0$

$\Rightarrow \boxed{x = (2n+1)\frac{\pi}{2}}$

CHAPTER-3 (MATRIX)

(1) $A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to 2R_2 - 5R_1$

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to 2R_3 - R_2$

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 2 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

~~$R_1 \to 5R_1 + R_2, \; R_3 \to 5R_3 - R_2$~~

$R_1 \to R_1 + R_3, \quad R_2 \to R_2 - 5R_3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -30 & 6 & -10 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_1 \to \frac{1}{2} R_1, \quad R_2 \to \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 3 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 3 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

(2) $$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 16+2x & 6+5x & 4+x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$

$\Rightarrow 16 + 2x + 12 + 10x + 4x + x^2 = 0$

$\Rightarrow$ ~~$x^2 + 11x + 22 = 0$~~

$x^2 + 16x + 28 = 0 \;\Rightarrow\; x = -14, \; x = -2$

③ Total amount

$= \cancel{A}BA$

$= \cancel{[40 \quad 100 \quad 50]}$

$$= \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 1000 \end{bmatrix} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 40000 + 50000 + 250000 \\ 120000 + 300000 + 50000 \end{bmatrix}$$

$$= \begin{bmatrix} 3,40,000 \\ 4,70,000 \end{bmatrix}$$

Total amount spent

$= 8,10,000$ Ans

④ $$A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & 6 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$ Ans

⑤ $$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - 5A + 7I = 0$$

$$A(A - 5I) = -7I$$

$$A^{-1} = \frac{1}{7}[5I - A]$$

$$A^{-1} = \frac{1}{7}\left[\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right]$$

$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \underline{A}$$

(6) $$\begin{bmatrix} 2\% & x\% \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix} = [1800]$$

$$2\% \text{ of } 15000 + x\% \text{ of } 15000 = 1800$$

$$\frac{2}{1\cancel{00}} \times 150\cancel{00} + \frac{x}{1\cancel{00}} \times 150\cancel{00} = 1800$$

$$150x = 1800 - 300$$

$$150x = 1500$$

$$x = 10\% \quad \underline{A}$$

(7) $$AA' = [a\ b\ c]\begin{bmatrix} a \\ b \\ c \end{bmatrix} = a^2+b^2+c^2$$

(8) $$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} ; \quad a_{ij} = (i-j)^3$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \underline{A}$$

CHAPTER-4 (DETERMINANT)

(1) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 \to R_1+R_2+R_3$

$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$C_1 \to C_1 - C_2, \quad C_2 \to C_2 - C_3$

$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ a+b+c & -(a+b+c) & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$

$= (a+b+c)^3 \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = (a+b+c)^3 (1-0)$

$= (a+b+c)^3$ R.H.S

(2) L.H.S $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

$= abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

$R_1 \to R_1+R_2+R_3$

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

$C_1 \to C_1 - C_2, \quad C_2 \to C_2 - C_3$

$= (abc+bc+ca+ab) \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1/b \\ 0 & -1 & \frac{1}{c}+1 \end{vmatrix} = (abc+ab+bc+ca)(1-0)$

$= ab+bc+ca+abc$

(3) Let for Honesty = Rs. x, for Regularity = Rs. y
for Hard Work = Rs. z.

$$x+y+z = 6000 \quad \text{—(1)}$$
$$x+3z = 11000 \quad \text{—(2)}$$
$$x+z = 2y \Rightarrow x-2y+z=0 \quad \text{—(3)}$$

$$AX = B$$
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$|A| = 1(0+6) - 1(1-3) + 1(-2-0) = 6+2-2 = 6$$

$$\text{adj}A = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}' = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 36000-33000+0 \\ 12000+0-0 \\ -12000+33000+0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix}$$

$$= \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

$$x = 500, \quad y = 2000, \quad z = 3500$$

(4)
$$\begin{vmatrix} 1+i & 1-i \\ 1-i & 1+i \end{vmatrix} = (1+i)^2 - (1-i)^2$$
$$= 1+i^2+2i-1-i^2+2i$$
$$= 4i \quad \text{A}$$

(5) L.H.S. $\begin{vmatrix} a^2 & abc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

$= abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ ab & b+c & c \end{vmatrix}$

$R_1 \to R_1 - (R_2+R_3)$

$= abc \begin{vmatrix} -2b & -2b & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

$= -2ab^2c \begin{vmatrix} -1 & -1 & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

$C_2 \to C_2 - C_1$

$= -2ab^2c \begin{vmatrix} -1 & 0 & 0 \\ a+b & -a & a \\ b & c & c \end{vmatrix} = -2ab^2c(-ac-ac)$
$= 4a^2b^2c^2$

(6) $\frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 35$

$\Rightarrow 2(4-4) + 6(5-k) + 1(20-4k) = 70$
$\Rightarrow 50 - 10k = 70 \Rightarrow \boxed{k = -2}$

(7) $AX = B$

$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$

$|A| = 3(2-3) + 2(4+4) + 3(-6-4) = -3 + 16 - 30$
$= -17$

$$\text{adj}A = \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & +1 \\ -1 & +9 & 7 \end{bmatrix}' = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= -\frac{1}{17}\begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1,\ y = 2,\ z = 3$

(9) $$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\Rightarrow (1+xyz)\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz)(x-y)(y-z)\begin{vmatrix} 1 & x+y \\ 1 & y+z \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz)(x-y)(y-z)(z-x) = 0$$

but $x \neq y \neq z$ $\therefore 1 + xyz = 0.$

Shaly 30/11

CHAPTER-5 (DIFFERENTIATION)

(1) $f(x) = \log_e(\log_e x)$

$f'(x) = \frac{1}{\log_e x} \cdot \frac{1}{x}$

$\therefore f'(e) = \frac{1}{\log_e e} \cdot \frac{1}{e} = 1 \times \frac{1}{e} = \frac{1}{e}$ Ans

(2) $\because f(x) = x+1$

$f \circ f(x) = f(x+1) = x+1+1 = x+2$

$\frac{d}{dx}\{f \circ f(x)\} = \frac{d}{dx}(x+2) = 1$ Ans

(3) $\because y = f(\log_e x)$

$\Rightarrow dy/dx = f'(\log_e x) \times \frac{1}{x}$

$\Rightarrow \frac{dy}{dx}_{(x=e)} = f'(\log_e e) \times \frac{1}{e}$

$= f'(1) \times \frac{1}{e} = \frac{2}{e}$ Ans

(4) $\because y = x|x|$

$y = x \times -x$, when $x < 0$

$y = -x^2$

$\frac{dy}{dx} = -2x$ Ans

(5) $y = x^x$

taking log on both sides

$\log y = x \cdot \log x$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{1}{x} + \log x\cdot 1$$

$$\Rightarrow \frac{dy}{dx} = y(1+\log x)$$

$$= x^{x}(1+\log x)$$

$$\left.\frac{dy}{dx}\right|_{(x=e)} = e^{e}(1+\log e) \qquad [\log e = 1]$$

$$= 2\cdot e^{e} \text{ Ans}$$

(6) $$y = \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$$

$$= \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2 x}{2\cos^2 x}}$$

$$y = \tan^{-1}(\tan x) \Rightarrow y = x$$

$$\Rightarrow \frac{dy}{dx} = 1 \text{ Ans}$$

(7) $$x = a(\theta + \sin\theta) \Rightarrow \frac{dx}{d\theta} = a(1+\cos\theta)$$

$$y = a(1-\cos\theta) \Rightarrow \frac{dy}{d\theta} = a(0+\sin\theta)$$
$$= a\sin\theta$$

$$dy/dx = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{\cancel{a}(1+\cos\theta)}$$

$$= \frac{\cancel{2}\sin\frac{\theta}{2}\cdot\cancel{\cos\frac{\theta}{2}}}{\cancel{2}\cos^{2}\frac{\theta}{2}} = \tan\frac{\theta}{2} \text{ Ans}$$

(8) $y = \sin^{-1} x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\cdot\dfrac{dy}{dx} = 1$

sq. both sides.

$(1-x^2)\left(\dfrac{dy}{dx}\right)^2 = 1$

$\Rightarrow (1-x^2)\cdot 2\dfrac{dy}{dx}\cdot\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 \times (-2x) = 0$

$\Rightarrow (1-x^2)\cdot\dfrac{d^2y}{dx^2} - x\cdot\dfrac{dy}{dx} = 0$ hp

---

(9) $y = (x\cdot\cos x)^x + (x\cdot\sin x)^{1/x}$

$y = u + v \Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ —(1)

$u = (x\cdot\cos x)^x$

$\log u = x\cdot\log(x\cos x)$

$\dfrac{1}{u}\cdot\dfrac{du}{dx} = x \times \dfrac{1}{x\cos x}(-x\sin x + \cos x) + \log(x\cos x)$

$= \dfrac{-x\sin x + \cos x}{\cos x} + \log(x\cos x)$

$\dfrac{du}{dx} = (x\cos x)^x\left[\dfrac{-x\sin x + \cos x}{\cos x} + \log(x\cdot\cos x)\right]$

$v = (x\sin x)^{1/x}$

$\log v = \dfrac{1}{x}\cdot\log(x\sin x)$

$\dfrac{1}{v}\cdot\dfrac{dv}{dx} = \dfrac{1}{x}\times\dfrac{1}{x\sin x}(x\cos x + \sin x) + \log(x\sin x)\times\dfrac{-1}{x^2}$

$\dfrac{dv}{dx} = (x\sin x)^{1/x}\left[\dfrac{x\cos x + \sin x}{x^2\sin x} - \dfrac{\log(x\sin x)}{x^2}\right]$

$\therefore$ from (1)

$$\frac{dy}{dx} = (x\cos x)^x\left[\frac{-x\sin x + \cos x}{\cos x} + \log(x\cos x)\right] + (x\sin x)^{1/x}\left[\frac{x\cos x + \sin x}{x^2\sin x} - \frac{\log(x\sin x)}{x^2}\right]$$

(10) $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$

put $x^3 = \sin\theta$; $y^3 = \sin\phi$

$\theta = \sin^{-1}x^3$; $\phi = \sin^{-1}y^3$

$$\sqrt{1-\sin^2\theta} + \sqrt{1-\sin^2\phi} = a(\sin\theta - \sin\phi)$$

$$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$$

$$\Rightarrow \cancel{2}\cancel{\cos\frac{\theta+\phi}{2}}\cdot\cos\frac{\theta-\phi}{2} = a\times\cancel{2}\cancel{\cos\frac{\theta+\phi}{2}}\sin\frac{\theta-\phi}{2}$$

$$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$$

$$\Rightarrow \frac{\theta-\phi}{2} = \cot^{-1}a \Rightarrow \theta - \phi = 2\cot^{-1}a$$

$$\Rightarrow \sin^{-1}x^3 - \sin^{-1}y^3 = 2\cot^{-1}a$$

$$\Rightarrow \frac{3x^2}{\sqrt{1-x^6}} - \frac{3y^2}{\sqrt{1-y^6}}\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}} \quad \text{hp}$$

(11) $y = \{\log(x+\sqrt{x^2+1})\}^2$

$$\Rightarrow y_1 = 2\log(x+\sqrt{x^2+1}) \times \left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right) \times \frac{1}{x+\sqrt{x^2+1}}$$

$$y_1 = 2\log(x+\sqrt{x^2+1}) \times \frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}} \times \frac{1}{\cancel{x+\sqrt{x^2+1}}}$$

$$\Rightarrow \sqrt{x^2+1}\cdot y_1 = 2\log(x+\sqrt{x^2+1})$$

sq. both sides

$$\Rightarrow (x^2+1)y_1^2 = 4\{\log(x+\sqrt{x^2+1})\}^2$$

$$\Rightarrow (x^2+1)y_1^2 = 4y$$

$$\Rightarrow (x^2+1)\cdot 2y_1y_2 + y_1^2\times 2x = 4y_1 \Rightarrow (x^2+1)y_2 + xy_1 = 2 \quad \text{hp}$$

CHAPTER-6 (APPLICATIONS OF DERIVATIVES)

(1) Let the edge of cube $= x$

$$\frac{dx}{dt} = 10 \text{ cm/sec}, \quad \frac{dv}{dt} = ?$$

$$V = x^3$$

$$\Rightarrow \frac{dv}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

$$= 3 \times 5 \times 5 \times 10$$

$$= 750 \text{ cm}^3/\text{sec. Ans}$$

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(2) $\sqrt{25.2}$

Let $y = \sqrt{x}$, where $x = 25$, $\Delta x = 0.2$

$$\Delta y = \frac{dy}{dx} \times \Delta x = \frac{1}{2\sqrt{x}} \times \Delta x$$

$$= \frac{1}{2\sqrt{25}} \times \Delta x = \frac{1}{10} \times 0.2$$

$$= 0.02$$

$$\cancel{y + \Delta y} = \sqrt{x + \Delta x} = y + \Delta y$$

$$= \sqrt{25} + 0.02$$

$$= 5.02 \text{ Ans}$$

---

(3) The curve is $x^2 + 3y + y^2 = 5$

$$\Rightarrow 2x + 3 \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = 0.$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2x}{3 + 2y}$$

$$\Rightarrow m = \frac{dy}{dx}_{(1,1)} = -\frac{2 \times 1}{3 + 2 \times 1}$$

$$= -\frac{2}{5} \text{ Ans}$$

(4) $f(x) = -|x-1| + 5$

$\because \forall x \in R, \quad |x-1| \geq 0$

$\Rightarrow -|x-1| \leq 0$

$\Rightarrow -|x-1| + 5 \leq 5$

$\Rightarrow f(x) \leq 5$

$\therefore$ max. value $= 5$ and there is no min. value.

(5) $\frac{dy}{dt} = 8 \cdot \frac{dx}{dt}$ (given).

$6y = x^3 + 2 \Rightarrow 6 \cdot \frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt}$

$\Rightarrow \overset{2}{\cancel{6}} \times 8 \cdot \frac{\cancel{dx}}{\cancel{dt}} = \cancel{3}x^2 \cdot \frac{\cancel{dx}}{dt}$

$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

when $x = 4$

$6y = 64 + 2 = 66 \Rightarrow y = 11$

when $x = -4$

$6y = -64 + 2 = -62 \Rightarrow y = -\frac{31}{3}$

$\therefore$ The required points are

$(4, 11)$ & $\left(-4, -\frac{31}{3}\right)$ Ans

(6) $f(x) = x^2 - 5x + 6$

(1) $f(x)$ being polynomial, $f(x)$ is cont. everywhere

(2) $f'(x) = 2x - 5$ satisfies for all values of $x \in (2,3)$

$\therefore f(x)$ is diff. in $(2,3)$

(3) $f(2) = 4 - 10 + 6 = 0$

$f(3) = 9 - 15 + 6 = 0.$

$f(2) = f(3)$

So all the three cond^{n}s of Rolle's th^{m} are satisfied.

So there exists a real no. $c \in (2,3)$, such that

$f'(c) = 0$

$\Rightarrow 2c - 5 = 0 \Rightarrow c = \frac{5}{2} \in (2,3)$

$\therefore$ Rolle's th^{m} is verified.

(7) $y = 5x^2 + 6x + 7$

$\frac{dy}{dx} = 10x + 6$

$m = \frac{dy}{dx}_{(1/2,\, 35/4)} = 10 \times \frac{1}{2} + 6$
$= 5 + 6 = 11$

The tangent is

$y - y_1 = m(x - x_1)$

$y - \frac{35}{4} = 11\left(x - \frac{1}{2}\right)$

$\Rightarrow \frac{4y - 35}{\cancel{4}\,2} = \frac{11(2x-1)}{\cancel{2}}$

$\Rightarrow 4y - 35 = 44x - 22$

$\Rightarrow \boxed{44x - 4y + 13 = 0}$ Ans

(8) Let (x_1, y_1) be the point of contact.

$\because 3x^2 - y^2 = 8$

$\Rightarrow 6x - 2y \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{6x}{2y} = \frac{3x}{y}$

m = slope of tangent $= \frac{3x_1}{y_1}$

$m_1 =$ slope of normal $= -\frac{1}{m} = -\frac{y_1}{3x_1}$

from the line $x+3y=4$

$$m_2 = -\frac{1}{3}$$

$$\because \quad m_1 = m_2$$

$$\Rightarrow -\frac{y_1}{3x_1} = -\frac{1}{3} \Rightarrow x_1 = y_1$$

also $3x_1^2 - y_1^2 = 8$

$$\Rightarrow 3x_1^2 - x_1^2 = 8 \Rightarrow x_1^2 = 4$$

$$\Rightarrow x_1 = \pm 2, \; y_1 = \pm 2$$

$\therefore$ The pts. of contact are $(2,2)$; $(-2,-2)$

The normals are $y - y_1 = -\frac{1}{m}(x - x_1)$

$y-2 = -\frac{1}{3}(x-2)$ & $y+2 = -\frac{1}{3}(x+2)$

$3y-6 = -x+2$ & $3y+6 = -x-2$

$\boxed{x+3y-8=0}$ & $\boxed{x+3y+8=0}$

---

(9)

$$f(x) = \sin x + \cos x$$

$$f'(x) = \cos x - \sin x$$

put $f'(x) = 0 \Rightarrow \cos x - \sin x = 0$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow x = \frac{\pi}{4} \text{ or } \pi + \frac{\pi}{4}$$

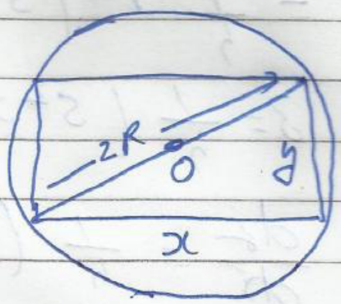
$$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\underset{0}{\vert} \overset{+}{\longrightarrow} \underset{\frac{\pi}{4}}{\vert} \overset{-}{\longrightarrow} \underset{\frac{5\pi}{4}}{\vert} \overset{+}{\longrightarrow} \underset{2\pi}{\vert}$$

$\therefore f(x)$ is inc. in $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

$f(x)$ is dec. in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ Ans

(10) $A = xy$



$A = x\sqrt{4R^2 - x^2}$

$A^2 = x^2(4R^2 - x^2)$

$z = 4R^2x^2 - x^4$

$\frac{dz}{dx} = 8R^2x - 4x^3$

put $dz/dx = 0$

$\Rightarrow 8R^2x - 4x^3 = 0 \Rightarrow x^2 = 2R^2$

$\Rightarrow x = R\sqrt{2}$

$y^2 = 4R^2 - x^2 \Rightarrow y^2 = 4R^2 - 2R^2 = 2R^2$

$\Rightarrow y = R\sqrt{2}$

$\frac{d^2z}{dx^2} = 8R^2 - 12x^2$

$= 8R^2 - 24R^2 < 0 \quad [x^2 = 2R^2]$

$\therefore$ Area is max. when $x = y$ i.e.
When rectangle is a square

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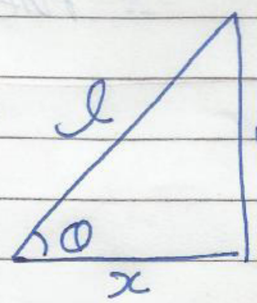
(11) $l + x = S$

$x = S - l \; ; \; l = S - x$



$A = \frac{1}{2}xy$

$A^2 = \frac{1}{4}x^2y^2$

$$z = \frac{1}{4} x^2 (l^2 - x^2)$$

$$= \frac{1}{4} x^2 \{ (S-x)^2 - x^2 \}$$

$$= \frac{1}{4} x^2 (S^2 + \cancel{x^2} - 2Sx - \cancel{x^2})$$

$$z = \frac{1}{4} (S^2 x^2 - 2S x^3)$$

$$\frac{dz}{dx} = \frac{1}{4} (2S^2 x - 6S x^2)$$

put $\frac{dz}{dx} = 0$

$$\cancel{2} S^{\cancel{2}} \cancel{x} = \overset{3}{\cancel{6}} \cancel{S} x^{\cancel{2}}$$

$$S = 3x$$

$$\Rightarrow l + x = 3x$$

$$\Rightarrow l = 2x$$

$$\Rightarrow \frac{x}{l} = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\frac{d^2 z}{dx^2} = \frac{1}{4} (2S^2 - 12Sx)$$

$$= \frac{1}{4} (18x^2 - 36x^2) < 0$$

$\therefore$ Area is max. when $\theta = \frac{\pi}{3}$.

CHAPTER- 7 (INTEGRATION)

(1) (a) $\int \frac{x^2+1}{x^4+1}dx$

$= \int \frac{1+\frac{1}{x^2}}{\left(x^2+\frac{1}{x^2}-2\right)+2}dx = \int \frac{1+1/x^2}{\left(x-\frac{1}{x}\right)^2+2}dx$ put $x-\frac{1}{x}=t$

$\Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

$= \int \frac{dt}{t^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)+c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)+c$ A

(b) $\int \frac{dx}{1+3\sin^2x+8\cos^2x}$

divide by $\cos^2x$

$\int \frac{\sec^2x\,dx}{\sec^2x+3\tan^2x+8}$

$= \int \frac{\sec^2x\,dx}{1+\tan^2x+3\tan^2x+8}$

$= \int \frac{\sec^2x\,dx}{4\tan^2x+9}$ put $\tan x = t$

$\sec^2x\,dx = dt$

$= \int \frac{dt}{4t^2+9} = \frac{1}{4}\int \frac{dt}{t^2+\left(\frac{3}{2}\right)^2}$

$= \frac{1}{4}\times\frac{2}{3}\tan^{-1}\left(\frac{2t}{3}\right)+c$

$= \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+c$ A

(2) $\int \frac{dx}{\sin x - \sin 2x}$

$= \int \frac{dx}{\sin x - 2\sin x \cdot \cos x}$

$= \int \frac{dx}{\sin x\,(1-2\cos x)}$

$= \int \frac{\sin x \, dx}{\sin^2 x\,(1-2\cos x)}$

$= \int \frac{\sin x \, dx}{(1-\cos^2 x)(1-2\cos x)}$

$= \int \frac{\sin x \, dx}{(1-\cos x)(1+\cos x)(1-2\cos x)}$

put $\cos x = t \Rightarrow -\sin x \, dx = dt$

$= \int \frac{-dt}{(1-t)(1+t)(1-2t)}$

$= -\int \frac{dt}{(t-1)(t+1)(2t-1)}$

Let $\frac{-1}{(t-1)(t+1)(2t-1)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{2t-1}$

$-1 = A(t+1)(2t-1) + B(t-1)(2t-1) + C(t-1)(t+1)$

put $t=1$

$-1 = A \times 2 \times 1$

$\boxed{A = -1/2}$

put $t=-1$

$-1 = B \times -2 \times -3$

$\boxed{B = -\frac{1}{6}}$

put $t = 1/2$

$-1 = C \times \frac{-1}{2} \times \frac{3}{2}$

$\boxed{C = \frac{4}{3}}$

$= -\frac{1}{2}\int \frac{dt}{t-1} - \frac{1}{6}\int \frac{dt}{t+1} + \frac{4}{3}\int \frac{dt}{2t-1}$

$= \frac{-1}{2} \log|t-1| - \frac{1}{6} \log|t+1| + \frac{4}{3} \times \frac{1}{2} \log|2t-1| + c$

$= -\frac{1}{2} \log|\cos x - 1| - \frac{1}{6} \log|\cos x + 1| + \frac{2}{3} \log|2\cos x - 1| + c$

(3) $\int \frac{\sqrt{1-\sin x}}{1+\cos x}\, e^{-x/2}\, dx$

$= \int \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{2\cos^2\frac{x}{2}} \cdot e^{-x/2}\, dx$

$= \frac{1}{2}\int e^{-x/2} \sec\frac{x}{2}\, dx - \frac{1}{2}\int e^{-x/2} \sec\frac{x}{2} \cdot \tan\frac{x}{2}\, dx$

$= \frac{1}{2}\left[\sec\frac{x}{2} \times -2e^{-x/2} - \int \frac{1}{2}\sec\frac{x}{2}\tan\frac{x}{2} \times -2e^{-x/2}\, dx\right]$

$- \frac{1}{2}\int e^{-x/2} \sec\frac{x}{2}\tan\frac{x}{2}\, dx$

$= -e^{-x/2}\sec\frac{x}{2} + \frac{1}{2}\int \cancel{e^{-x/2}\sec\frac{x}{2}\tan\frac{x}{2}\, dx}$

$- \frac{1}{2}\int e^{-x/2}\cancel{\sec\frac{x}{2}}\tan\frac{x}{2}\, dx$

$= -e^{-x/2}\sec\frac{x}{2} + c$ Ans

(4) $\int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

$= \int_0^{\pi/2} \left(\sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}}\right) dx$

$= \int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{\sin x \cdot \cos x}}\, dx$

$= \sqrt{2}\int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

$= \sqrt{2}\int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{1-(1-\sin 2x)}}\, dx$

$= \sqrt{2}\int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}}\, dx$

Put $\sin x - \cos x = t$
$\Rightarrow (\cos x + \sin x)\,dx = dt$
$x \to 0 \Rightarrow t \to -1$
$x \to \frac{\pi}{2} \Rightarrow t \to 1$

$= \sqrt{2}\int_{-1}^{1} \frac{dt}{\sqrt{1-t^2}}$

$= \sqrt{2}\left[\sin^{-1} t\right]_{-1}^{1} = \sqrt{2}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \pi\sqrt{2}$ Ans

⑤ $I = \int_0^{\pi} \frac{x \cdot \tan x}{\sec x + \tan x}\, dx$ —— (1)

$= \int_0^{\pi} \frac{(\pi - x) \cdot \tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)}\, dx$

$= \int_0^{\pi} \frac{(\pi - x) \cdot (-\tan x)}{-\sec x - \tan x}\, dx$

$I = \int_0^{\pi} \frac{(\pi - x) \cdot \tan x}{\sec x + \tan x}\, dx$ —— (2)

$2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x}\, dx$

$= \pi \int_0^{\pi} \frac{\sec x \tan x - \tan^2 x}{\sec^2 x - \tan^2 x}\, dx$

$= \pi \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1)\, dx$

$= \pi \left[\sec x - \tan x + x \right]_0^{\pi}$

$= \pi \left[(\sec \pi - \tan \pi + \pi) - (\sec 0 - \tan 0 + 0) \right]$

$= \pi \left[-1 - 0 + \pi - 1 + 0 - 0 \right]$

$2I = \pi (\pi - 2)$

$I = \pi \left(\frac{\pi}{2} - 1 \right)$ Ans

⑥ $I = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}\, dx$ —— (1)

$= \int_0^{\pi/2} \frac{\sin^n \left(\frac{\pi}{2} - x\right)}{\sin^n \left(\frac{\pi}{2} - x\right) + \cos^n \left(\frac{\pi}{2} - x\right)}\, dx$

$$I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx \quad \text{———} (2)$$

adding (1) & (2)

$$2I = \int_0^{\pi/2} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx$$

$$2I = \int_0^{\pi/2} 1 \cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4} \text{ Ans}$$

(7) $\int \frac{dx}{x^3(x^5+1)^{3/5}} dx$

$$= \int \frac{dx}{x^6\left(1+\frac{1}{x^5}\right)^{3/5}} = \int \frac{x^{-6}}{\left(1+\frac{1}{x^5}\right)^{3/5}} dx$$

put $1+\frac{1}{x^5} = t$

$-\frac{5}{x^6} dx = dt$

$$= -\frac{1}{5}\int \frac{dt}{t^{3/5}}$$

$$= -\frac{1}{5}\int t^{-3/5} dt = -\frac{1}{5} \times \frac{t^{2/3}}{\frac{2}{3}} + c$$

$$= -\frac{3}{10}\left(1+\frac{1}{x^5}\right)^{2/3} + c \text{ Ans}$$

(8) $\int \frac{x^4}{(x-1)(x^2+1)} dx$

$$= \int \frac{x^4}{x^3 - x^2 + x - 1} dx$$

$$\begin{array}{r|l} x^3 - x^2 + x - 1 \;) \; x^4 & (\; x+1 \\ x^4 - x^3 + x^2 - x & \\ \underline{- \quad + \quad - \quad +} & \\ x^3 - x^2 + x & \\ x^3 - x^2 + x - 1 & \\ \underline{- \quad + \quad - \quad +} & \\ 1 & \end{array}$$

$$I = \int (x+1) + \frac{1}{(x-1)(x^2+1)} dx \quad \text{———} (1)$$

Let
$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + (Bx+C)(x-1)$$

$$1 = Ax^2 + A + Bx^2 - Bx + Cx - C$$

$$1 = x^2(A+B) + x(-B+C) + (A-C)$$

$$A+B=0, \quad -B+C=0, \quad A-C=1$$

put $x=1$; $1 = A(1+1) \Rightarrow \boxed{A=\frac{1}{2}}$

$B=-A \Rightarrow \boxed{B=-\frac{1}{2}}$; $C=B$

$\boxed{C=-\frac{1}{2}}$

from (I)

$$I = \int x\,dx + \int dx + \frac{1}{2}\int\frac{dx}{x+1} + \int\frac{-\frac{1}{2}x-\frac{1}{2}}{x^2+1}\,dx$$

$$= \int x\,dx + \int dx + \frac{1}{2}\int\frac{dx}{x+1} - \frac{1}{4}\int\frac{2x}{x^2+1}\,dx - \frac{1}{2}\int\frac{dx}{x^2+1}$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log|x+1| - \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + C$$

(9) $\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ put $\sqrt{x} = \cos\theta$

$x = \cos^2\theta$

$dx = -2\sin\theta\cdot\cos\theta\,d\theta$

$$= \int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \times -2\sin\theta\cdot\cos\theta\,d\theta$$

$$= \int\sqrt{\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\cos^2\theta/2}} \times -2\sin\theta\cdot\cos\theta\,d\theta$$

$$= \int\frac{\sin\theta/2}{\cancel{\cos\theta/2}} \times -2\times 2\sin\frac{\theta}{2}\cdot\cancel{\cos\frac{\theta}{2}}\cdot\cos\theta\,d\theta$$

$$= -2\int 2\sin^2\frac{\theta}{2}\cdot\cos\theta\,d\theta$$

$$= -2\int(1-\cos\theta)\cdot\cos\theta\,d\theta$$

$$= \int (2\cos^2\theta - 2\cos\theta)\, d\theta$$

$$= \int (1 + \cos 2\theta - 2\cos\theta)\, d\theta$$

$$= \theta + \frac{\sin 2\theta}{2} - 2\sin\theta + c$$

$$= \theta + \frac{\cancel{2}\sin\theta\cdot\cos\theta}{\cancel{2}} - 2\sin\theta + c$$

$$= \theta + \sqrt{1-\cos^2\theta}\cdot\cos\theta - 2\sqrt{1-\cos^2\theta} + c$$

$$= \cos^{-1}\sqrt{x} + \sqrt{1-x}\cdot\sqrt{x} - 2\sqrt{1-x} + c \quad \text{Ans}$$

(10) $\int (x-2)\sqrt{x^2+3x-18}\, dx$

$x-3 = l(2x+3) + m$

$x-3 = 2lx + 3l + m$

$2l = 1 \Rightarrow \boxed{l = 1/2}$; $3l + m = -3$

$\frac{3}{2} + m = -3$

$m = -3 - \frac{3}{2} \Rightarrow \boxed{m = \frac{-9}{2}}$

$$\int \left\{\frac{1}{2}(2x+3) - \frac{9}{2}\right\} \sqrt{x^2+3x-18}\, dx$$

$$= \frac{1}{2}\int (2x+3)\sqrt{x^2+3x-18}\, dx - \frac{9}{2}\int \sqrt{\left(x^2 + 2\cdot x\cdot\frac{3}{2} + \frac{9}{4}\right) - 18 - \frac{9}{4}}\, dx$$

$$= \text{———} - \frac{9}{2}\int \sqrt{\left(x+\frac{3}{2}\right)^2 - \frac{81}{4}}\, dx$$

$$= \text{———} - \frac{9}{2}\int \sqrt{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\, dx$$

$$= \frac{1}{2}\times\frac{2}{3}(x^2+3x-18)^{3/2} - \frac{9}{2}\left[\frac{2x+3}{4}\sqrt{x^2+3x-18} - \frac{81}{8}\ln\left|\left(x+\frac{3}{2}\right) + \sqrt{x^2+3x-18}\right|\right] + c$$

Ans

(11) $\int_2^5 |x-2|+|x-3|+|x-5|\,dx$

$2 \le x < 3$

$f(x) = +(x-2)-(x-3)-(x-5)$

$= \cancel{x}-2-\cancel{x}+3-x+5 = 6-x$

$3 < x \le 5$

$f(x) = (x-2)+(x-3)-(x-5)$

$= x-2+\cancel{x}-3-\cancel{x}+5 = x$

$= \int_2^3 (6-x)\,dx + \int_3^5 x\,dx$

$= \left[6x - \frac{x^2}{2}\right]_2^3 + \frac{1}{2}\left[x^2\right]_3^5$

$= \left[\left(18-\frac{9}{2}\right)-(12-2)\right] + \frac{1}{2}(25-9)$

$= \frac{27}{2} - 10 + 8 = \frac{27}{2} - 2 = \frac{23}{2}$ A

(12) $\int_1^4 (x^2 - x)\,dx$

Here $a=1$, $b=4$, $nh = b-a$

$nh = 4-1 = 3$

$f(x) = x^2 - x$

$f(1) = 1-1 = 0$

$f(1+h) = (1+h)^2 - (1+h)$

$= 1+h^2+2h-1-h = h+h^2$

$f(1+2h) = (1+2h)^2 - (1+2h)$

$= 1+4h^2+4h-1-2h = 2h+4h^2$

— - —

— —

$$= \lim_{h \to 0} h\left[f(1) + f(1+h) + f(1+2h) + \text{--- n times}\right]$$

$$= \lim_{h \to 0} h\left[0 + (h+h^2) + (2h+4h^2) + \text{--- n times}\right]$$

$$= \lim_{h \to 0} h\left[h(1+2+3+\text{-- n-1 times}) + h^2(1^2+2^2+\text{-- n-1 times})\right]$$

$$= \lim_{h \to 0} h\left[h \cdot \frac{n(n-1)}{2} + h^2 \cdot \frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{h \to 0}\left[\frac{nh(nh-h)}{2} + \frac{nh(nh-h)(2nh-h)}{6}\right]$$

put $nh = 3$

$$= \lim_{h \to 0}\left[\frac{3(3-h)}{2} + \frac{3(3-h)(6-h)}{6}\right]$$

put $h = 0$

$$\frac{9}{2} + \frac{3 \times 3 \times 6}{6}$$

$$= \frac{9}{2} + 9$$

$$= \frac{27}{2} \quad A$$

(13) $\displaystyle\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$

$f(x) = \sin^7 x$

$f(-x) = \sin^7(-x) = -\sin^7 x$

$= -f(x)$

$= f(x)$ is odd.

$$\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx = 0 \quad A$$

CHAPTER – 8 (APPLICATIONS OF INTEGRALS)

① $x^2 = 4y$

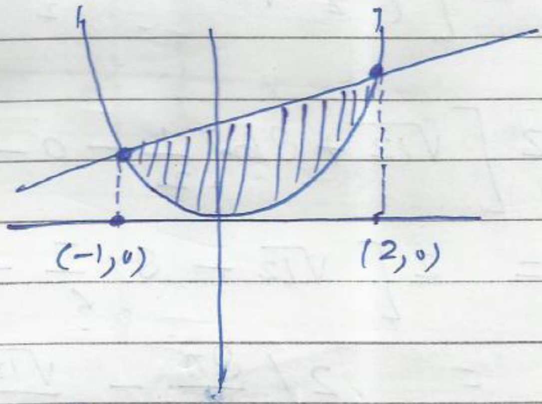
$x = 4y - 2 \Rightarrow 4y = x + 2$

$\therefore x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2, x = -1$



Req. Area

$$= \frac{1}{4}\int_{-1}^{2} (x+2)\,dx - \frac{1}{4}\int_{-1}^{2} x^2\,dx$$

$$= \frac{1}{4}\left(\frac{x^2}{2} + 2x\right)_{-1}^{2} - \frac{1}{12}\left(x^3\right)_{-1}^{2}$$

$$= \frac{1}{4}\left[(2+4) - \left(+\frac{1}{2} - 2\right)\right] - \frac{1}{12}(8+1)$$

$$= \frac{1}{4}\left[6 + \frac{3}{2}\right] - \frac{9}{12}$$

$$= \frac{15}{8} - \frac{3}{4} = \frac{15-6}{8} = \frac{9}{8} \text{ sq. units}$$

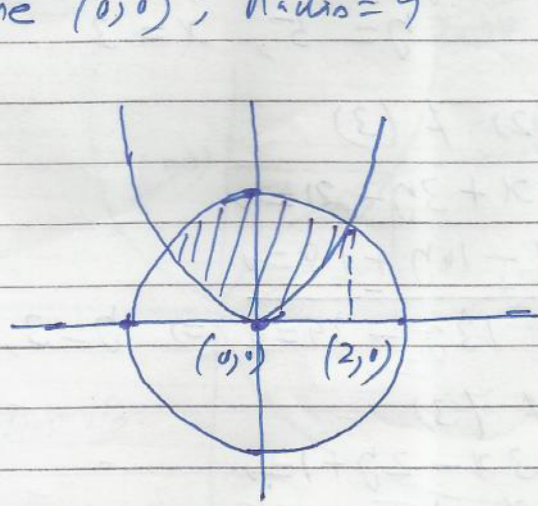
② $x^2 + y^2 = 16$; centre $(0,0)$; Radius $= 4$

$y^2 = 6x$

$\therefore x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8, x = 2$



$\frac{144}{19}$
$\overline{125}$

Req. Area

$$= 2\left[\int_0^2 \sqrt{16-x^2}\,dx - \int_0^2 \sqrt{6x}\,dx\right]$$

$$= 2\left[\left\{\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right\}_0^2 - \sqrt{6}\times\frac{2}{3}\left(x^{3/2}\right)_0^2\right]$$

$$= 2\left[\sqrt{12} + 8\sin^{-1}\frac{1}{2} - 0 - 0 - \frac{2\sqrt{6}}{3}\times 2\sqrt{2}\right]$$

$$= 2\left[\sqrt{12} + 8\times\frac{\pi}{6} - \frac{4\sqrt{12}}{3}\right]$$

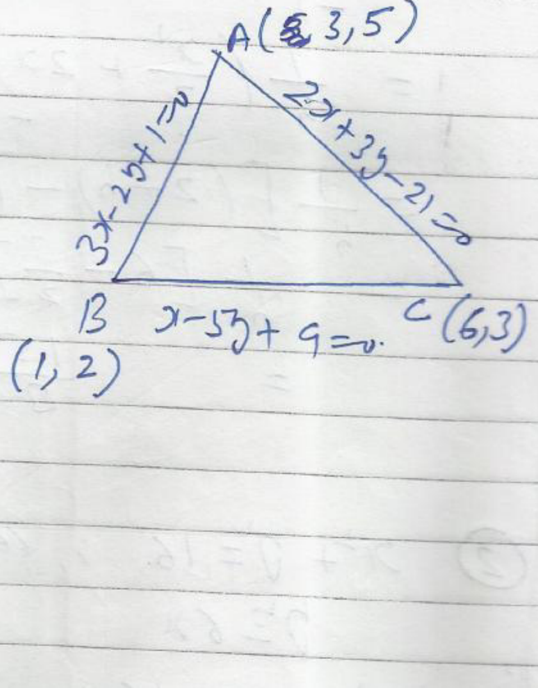
$$= 2\left(\frac{8\pi}{6} - \frac{\sqrt{12}}{3}\right)$$

$$= \left(\frac{8\pi}{3} - \frac{4\sqrt{3}}{3}\right) \text{ sq. units Ans}$$

(3) $3x-2y+1=0$ —(1)

$2x+3y-21=0$ —(2)

$x-5y+9=0$ —(3)



from (1) & (2).

$6x-4y+2=0$

$\underline{\overset{-}{}6x\underset{-}{+}9y\underset{+}{-}63=0}$

$-13y+65=0$

$y=5,\ x=3$

from (2) & (3)

$2x+3y-21=0$

$\underline{\overset{-}{}2x\underset{+}{-}10y\underset{-}{+}18=0}$

$13y-39=0 \Rightarrow y=3,\ x=6$

from (1) & (3)

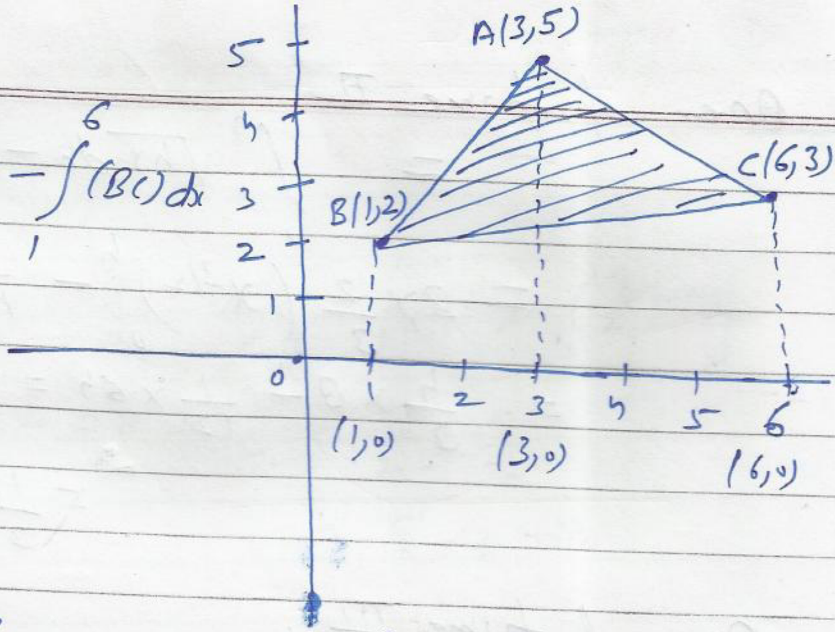
$3x-2y+1=0$

$\underline{\overset{-}{}3x\underset{+}{-}15y\underset{-}{+}27=0}$

$13y-26=0 \Rightarrow y=2,\ x=1$

Req. Area =

$$\int_1^3 (AB)\,dx + \int_3^6 (AC)\,dx - \int_1^6 (BC)\,dx$$



$$= \int_1^3 \frac{3x+1}{2}\,dx + \int_3^6 \frac{21-2x}{3}\,dx - \int_1^6 \frac{x+9}{5}\,dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2} + x\right]_1^3 + \frac{1}{3}\left[21x - x^2\right]_3^6 - \frac{1}{5}\left[\frac{x^2}{2} + 9x\right]_1^6$$

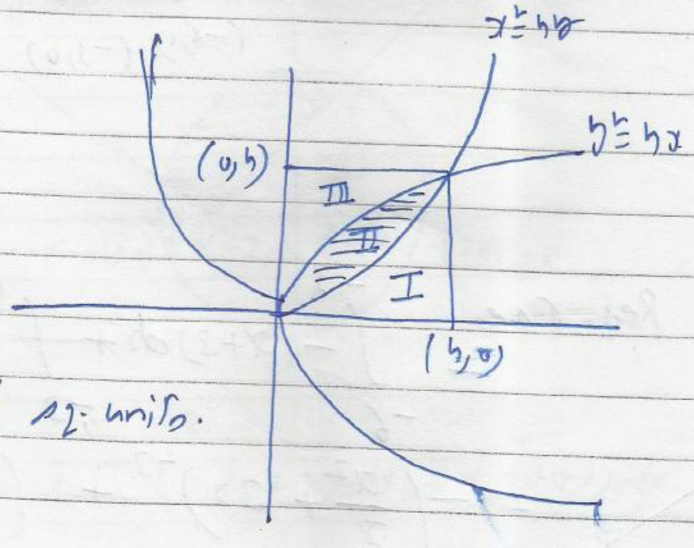
$$= \frac{1}{2}\left[\left(\frac{27}{2}+3\right) - \left(\frac{3}{2}+1\right)\right] + \frac{1}{3}\left[(126-36) - (63-9)\right] - \frac{1}{5}\left[(18+54) - \left(\frac{1}{2}+9\right)\right]$$

$$= \frac{1}{2}\left(\frac{33}{2} - \frac{5}{2}\right) + \frac{1}{3}(90-54) - \frac{1}{5}\left(72 - \frac{19}{2}\right)$$

$$= \frac{1}{\cancel{2}} \times \frac{28^{7}}{\cancel{2}} + \frac{1}{3} \times 36 - \frac{1}{5} \times \frac{125}{2}$$

$$= 7 + 12 - \frac{25}{2} = 19 - \frac{25}{2} = \frac{13}{2} \text{ sq. units.}$$

(4). Area of figure I



$$= \int_0^4 \frac{x^2}{4}\,dx$$

$$= \frac{1}{4} \times \frac{1}{3}\left(x^3\right)_0^4$$

$$= \frac{1}{12} \times 64 = \frac{16}{3} \text{ sq. units.}$$

Area of figure II

$$= \int_0^4 \sqrt{4x}\, dx - \int_0^4 \frac{x^2}{4} dx$$

$$= 2 \times \frac{2}{3}\left(x^{3/2}\right)_0^4 - \frac{1}{12}\left(x^3\right)_0^4$$

$$= \frac{4}{3} \times 8 - \frac{1}{12} \times 64 = \frac{32}{3} - \frac{16}{3}$$

$$= \frac{16}{3} \text{ sq. units.}$$

Area of figure III

$$= \int_0^4 \frac{y^2}{4} dy = \frac{1}{4} \times \frac{1}{3}\left[y^3\right]_0^4$$

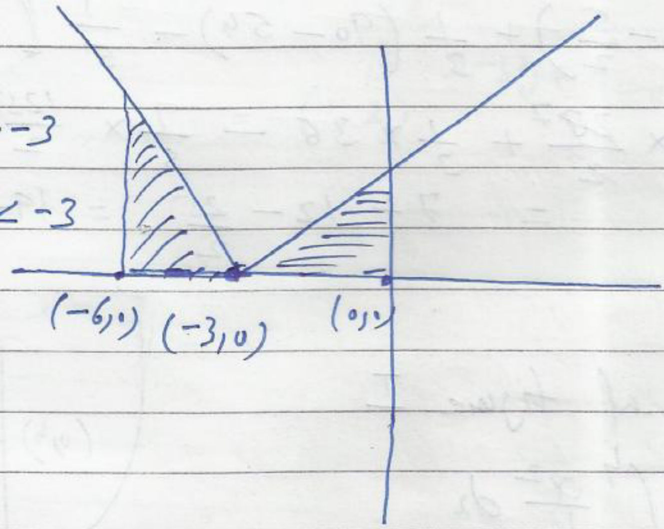
$$= \frac{1}{12} \times 64$$

$$= \frac{16}{3} \text{ sq. units.}$$

$$\therefore \text{ar.}(I) = \text{ar.}(II) = \text{ar.}(III) = \frac{16}{3} \text{ sq units}$$

⑤ $y = |x+3|$

$$y = \begin{cases} x+3, & x \geq -3 \\ -(x+3), & x < -3 \end{cases}$$



Req. Area

$$= \int_{-6}^{-3} -(x+3)\, dx + \int_{-3}^{0} (x+3)\, dx$$

$$= -\left(\frac{x^2}{2} + 3x\right)_{-6}^{-3} + \left(\frac{x^2}{2} + 3x\right)_{-3}^{0}$$

$$= -\left[\left(\frac{9}{2}-9\right)-\left(\frac{36}{2}-18\right)\right]+\left[(0+0)-\left(\frac{9}{2}-9\right)\right]$$

$$= -\left(-\frac{9}{2}-0\right)+\frac{9}{2} = \frac{9}{2}+\frac{9}{2}$$

$$= 9 \text{ sq-units.}$$

⑥ $\{(x,y) : |x+2| \le y \le \sqrt{20-x^2}\}$.

$y \geq |x+2|$

$y = x+2, \quad x \geq -2 \Rightarrow x-y+2=0, \; x \geq -2$

$y = -(x+2), \; x < -2 \Rightarrow x+y+2 \; ; \; x < -2$

$y = \sqrt{20-x^2}$

$x^2+y^2=20$ is a circle with centre $(0,0)$, Radi $=\sqrt{20}$

$x^2+(x+2)^2=20$

~~$x^2+x-18=0$~~

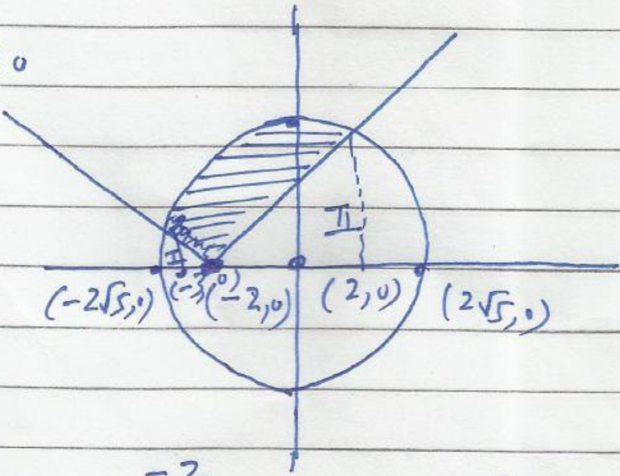
$x^2+x^2+4+4x=20$

$2x^2+4x-16=0$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4, \; x=2$



Req. Area

$$= \int_{-4}^{2}\sqrt{20-x^2}\,dx - \int_{-4}^{-2}-(x+2)\,dx - \int_{-2}^{2}(x+2)\,dx$$

Area of figure I

$= \oint$

$$= \left[\frac{x}{2}\sqrt{20-x^2} + \frac{20}{2}\sin^{-1}\frac{x}{\sqrt{20}}\right]_{-4}^{2} + \left[\frac{x^2}{2}+2x\right]_{-4}^{-2}$$

$$- \left[\frac{x^2}{2}+2x\right]_{-2}^{2}$$

$$= \left(\sqrt{16} + 10\sin^{-1}\frac{2}{\sqrt{20}}\right) - \left(-2\sqrt{20-16} + 10\sin^{-1}\frac{-4}{\sqrt{20}}\right)$$

$$+ \left[(2-4)-(8-8)\right] - \left[(2+4)-(2-4)\right]$$

$$= 4 + 10\sin^{-1}\frac{1}{\sqrt{5}} + 4 + 10\sin^{-1}\frac{2}{\sqrt{5}} - 2 - 8$$

$$= \left(10\sin^{-1}\frac{1}{\sqrt{5}} + 10\sin^{-1}\frac{2}{\sqrt{5}} - 2\right) \text{ sq-units}$$

CHAPTER-9 (DIFFERENTIAL EQUATION)

(1) order = 2, degree = 3

(2) $y = x\cdot\sin x \Rightarrow y' = x\cdot\cos x + \sin x$

L.h.s. $xy' = x(x\cos x + \sin x) = x^2\cos x + x\sin x$

R.h.s. $y + x\sqrt{x^2 - y^2}$

$= x\cdot\sin x + x\sqrt{x^2 - x^2\sin^2 x} = x\sin x + x\cdot x\cos x$

$= x^2\cos x + x\sin x = xy'$

(3) $\frac{x}{a} + \frac{y}{b} = 1$ —— (1)

Diff. (1) w.r.t. x

$\frac{1}{a} + \frac{1}{b}\cdot\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{b}{a}$ —— (2)

Again diff (2) w.r.t. x

$\frac{d^2y}{dx^2} = 0$ Ans

(4) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \Rightarrow \int\frac{dy}{1+y^2} = \int\frac{dx}{1+x^2} + c$

$\Rightarrow \tan^{-1}y - \tan^{-1}x = c$

$\Rightarrow \tan^{-1}\left(\frac{y-x}{1+xy}\right) = c \Rightarrow \frac{y-x}{1+xy} = c$

$\Rightarrow y - x = c(1+xy)$ Ans

(5) $x\cdot dy = (2x^2+1)dx$

$\Rightarrow \int dy = \int\frac{2x^2+1}{x}dx + c$

$\Rightarrow \int dy = \int\left(2x + \frac{1}{x}\right)dx + c$

$\Rightarrow y = x^2 + \log|x| + c$

put $x=1, y=1 \Rightarrow 1 = 1 + \log 1 + c$

$\Rightarrow c = 0.$

$\therefore$ The curve is

$y = x^2 + \log|x|$ Ans

⑥ $(x^3+x^2+x+1)\cdot\frac{dy}{dx} = 2x^2+x,$

$\Rightarrow \int dy = \int \frac{2x^2+x}{x^3+x^2+x+1}\,dx + C$

$\Rightarrow \int dy = \int \frac{2x^2+x}{(x+1)(x^2+1)}\,dx + C$ ——— (1)

$\frac{2x^2+x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$2x^2+x = A(x^2+1) + (Bx+C)(x+1)$

$= Ax^2 + A + Bx^2 + Bx + Cx + C$

$2x^2+x = x^2(A+B) + x(B+C) + (A+C)$

$A+B=2$; $B+C=1$, $A+C=0$

put $x=-1$

$2-1 = A(1+1) \Rightarrow \boxed{A=1/2}$; $C=-A$

$B=2-A \Rightarrow B = 2-\frac{1}{2} \Rightarrow \boxed{B=3/2}$ $\boxed{C=-\frac{1}{2}}$

from (1)

$\int dy = \frac{1}{2}\int\frac{dx}{x+1} + \int\frac{\frac{3}{2}x-\frac{1}{2}}{x^2+1}\,dx + C$

$\int dy = \frac{1}{2}\int\frac{dx}{x+1} + \frac{3}{4}\int\frac{2x}{x^2+1}\,dx - \frac{1}{2}\int\frac{dx}{x^2+1} + C$

$\Rightarrow y = \frac{1}{2}\ln|x+1| + \frac{3}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + C$

put $x=0, y=0$

$0 = 0+0-0+C \Rightarrow C=0.$

$\therefore y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x$

⑦ $\log\left(\frac{dy}{dx}\right) = 3x+4y \Rightarrow \frac{dy}{dx} = e^{3x+4y} = e^{3x}\cdot e^{4y}$

$\Rightarrow \int e^{-4y}\,dy = \int e^{3x}\,dx + C$

$\Rightarrow -\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} + C$

put $x=0, y=0 \Rightarrow -\frac{1}{4} = \frac{1}{3} + C \Rightarrow C = -\frac{7}{12}$

$\therefore -\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} - \frac{7}{12}$

⑧ $(x-y)\frac{dy}{dx} = x+2y$

$dy/dx = \frac{x+2y}{x-y}$; put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$= v + x\frac{dv}{dx} = \frac{x+2vx}{x-vx}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+2v-v+v^2}{1-v}$

$x\frac{dv}{dx} = \frac{v^2+v+1}{-(v-1)}$

$\Rightarrow \int \frac{v-1}{v^2+v+1}\,dv = -\int \frac{dx}{x} + c$

$\Rightarrow \frac{1}{2}\int \frac{2v+1-3}{v^2+v+1}\,dv + \int \frac{dx}{x} = c$

$\Rightarrow \int \frac{2v+1}{v^2+v+1}\,dv - 3\int \frac{dv}{\left(v^2+2\cdot v\cdot\frac{1}{2}+\frac{1}{4}\right)+\left(1-\frac{1}{4}\right)} + 2\int\frac{dx}{x} = c$

$\Rightarrow \int \frac{2v+1}{v^2+v+1}\,dv - 3\int \frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + 2\int\frac{dx}{x} = c$

$\Rightarrow \log|v^2+v+1| - 3\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) + 2\log x = c$

$\Rightarrow \log x^2\left(\frac{y^2}{x^2}+\frac{y}{x}+1\right) - 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2y}{x}+1}{\sqrt{3}}\right) = c$

$\Rightarrow \log(y^2+xy+x^2) - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}\,x}\right) = c$ Ans

⑨ $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} + \frac{1}{\sqrt{x}}\cdot y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$P = 1/\sqrt{x}$; $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

I.F. $= e^{\int P\,dx} = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$

Gen. soln is:

$y\cdot e^{2\sqrt{x}} = \int \frac{e^{-2\sqrt{x}}}{\sqrt{x}}\cdot e^{2\sqrt{x}}\,dx + c$

$ye^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}}\,dx + c \Rightarrow ye^{2\sqrt{x}} = 2\sqrt{x} + c$ A

(10) $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0 \Rightarrow \int \frac{dy}{y^2+y+1} + \int \frac{dx}{x^2+x+1} = c$

$\Rightarrow \int \frac{dy}{(y+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} + \int \frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} = c$

$\Rightarrow \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = c$

$\Rightarrow \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = c$

$\Rightarrow \tan^{-1}\left\{\frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1 - \frac{2y+1}{\sqrt{3}} \times \frac{2x+1}{\sqrt{3}}}\right\} = c$

$\Rightarrow \frac{\sqrt{3}\left[(2y+1)+(2x+1)\right]}{3-(2x+1)(2y+1)} = c$

$\Rightarrow \frac{2x+2y+2}{3-4xy-2x-2y-1} = c \Rightarrow \frac{x+y+1}{1-x-y-2xy} = c$

$\Rightarrow x+y+1 = c(1-x-y-2xy)$ Ans

---

(11) $\frac{dv}{dt} = k$

$\because v = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = \frac{4\pi}{3} \times 3r^2 \cdot \frac{dr}{dt}$

$\Rightarrow \frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$\therefore 4\pi r^2 \cdot \frac{dr}{dt} = k \Rightarrow \int r^2 \cdot dr = \frac{k}{4\pi}\int dt + c$

$\Rightarrow \frac{r^3}{3} = \frac{kt}{4\pi} + c$

Initially, $t=0$, $r=3$

$\therefore 9 = 0 + c \Rightarrow c = 9$

$\therefore \frac{r^3}{3} = \frac{kt}{4\pi} + 9$

$t=3$, $r=6$ $\quad \therefore \frac{216}{3} = 72 = \frac{3k}{4\pi} + 9$

$\frac{3k}{4\pi} = 72 - 9 = 63$

$\frac{k}{4\pi} = 21$

$\therefore \frac{r^3}{3} = 21t + 9 \Rightarrow \boxed{r = \sqrt[3]{63t+27}}$ Ans

CHAPTER-10 (VECTORS)

(1) $(2\hat{i}+6\hat{j}+14\hat{k})\times(\hat{i}-\lambda\hat{j}+7\hat{k})=\vec{0}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & +14 \\ 1 & -\lambda & 7 \end{vmatrix} = 0$$

$\Rightarrow \hat{i}(42-14\lambda)-\hat{j}(14-14)+\hat{k}(+2\lambda-6)=0$

$\Rightarrow \quad 42-14\lambda=0$

$\Rightarrow \quad \boxed{\lambda=3}$

(2) $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=4 \Rightarrow \dfrac{2\lambda+6+12}{\sqrt{4+36+9}}=4$

$\Rightarrow \quad 2\lambda+18=28$

$\Rightarrow \quad 2\lambda=10$

$\Rightarrow \quad \boxed{\lambda=5}$

(3) $\because \vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}$ are coplanar

$\therefore [\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}]=0$

$\Rightarrow (\vec{a}+\vec{b})\cdot\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\}=0$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a})=0$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c}-\vec{a}\times\vec{b}+0+\vec{c}\times\vec{a})=0$

$\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c})-\vec{a}\cdot(\vec{a}\times\vec{b})+\vec{a}\cdot(\vec{c}\times\vec{a})$

$\quad +\vec{b}\cdot(\vec{b}\times\vec{c})-\vec{b}\cdot(\vec{a}\times\vec{b})+\vec{b}\cdot(\vec{c}\times\vec{a})=0$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]-0+0+0-0+[\vec{a}\ \vec{b}\ \vec{c}]=0$

$\Rightarrow \quad 2[\vec{a}\ \vec{b}\ \vec{c}]=0 \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]=0$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are coplanar.

(4) $\vec{r_1} = \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$, $\vec{r_2} = \vec{a} - \vec{b}$

$= 2\hat{i} + 4\hat{k}$

Any vector $\perp^r$ to $\vec{r_1}$ & $\vec{r_2}$

$= \vec{r_1} \times \vec{r_2}$

$$\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$= \hat{i}(16-0) - \hat{j}(16-0) + \hat{k}(0-8)$

$= 16\hat{i} - 16\hat{j} - 8\hat{k}$

$\hat{d} = \dfrac{\vec{d}}{|\vec{d}|} = \dfrac{2\hat{i} - 2\hat{j} - \hat{k}}{\sqrt{5}}$ Ans

(5) $\because \vec{a} \parallel \vec{x} \Rightarrow \vec{a} = \lambda\vec{x} \Rightarrow \vec{a} = \lambda(3\hat{i} + 4\hat{j} + 5\hat{k})$

$\vec{y} = \vec{a} + \vec{b} \Rightarrow 2\hat{i} + \hat{j} - 4\hat{k} = \lambda(3\hat{i} + 4\hat{j} + 5\hat{k}) + \vec{b}$

$\Rightarrow \vec{b} = (2 - 3\lambda)\hat{i} + (1 - 4\lambda)\hat{j} - (4 + 5\lambda)\hat{k}$

$\because \vec{b} \perp \vec{a} \Rightarrow 3(2 - 3\lambda) + 4(1 - 4\lambda) - 5(4 + 5\lambda) = 0$

$\Rightarrow -10 - 50\lambda = 0 \Rightarrow \lambda = -\dfrac{1}{5}$

$\therefore \vec{b} = \left(2 + \dfrac{3}{5}\right)\hat{i} + \left(1 + \dfrac{4}{5}\right)\hat{j} - (4 - 1)\hat{k}$

$= \dfrac{13}{5}\hat{i} + \dfrac{9}{5}\hat{j} - 3\hat{k}$

& $\vec{a} = -\dfrac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k})$

$\therefore 2\hat{i} + \hat{j} - 4\hat{k} =$

$\left(-\dfrac{3}{5}\hat{i} - \dfrac{4}{5}\hat{j} - \hat{k}\right) + \lambda\left(\dfrac{13}{5}\hat{i} + \dfrac{9}{5}\hat{j} - 3\hat{k}\right).$

⑥ Any vector perp. to both $\vec{a}$ & $\vec{b}$

$$= \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$$= \hat{i}(28+4) - \hat{j}(7-6) + \hat{k}(-2-12)$$

$$= 32\hat{i} - \hat{j} - 14\hat{k}$$

but $\vec{p}$ is $\perp^r$ to both $\vec{a}$ & $\vec{b}$

$$\therefore \vec{p} \parallel \vec{a}\times\vec{b} \Rightarrow \vec{p} = \lambda(\vec{a}\times\vec{b})$$

$$= \lambda(32\hat{i} - \hat{j} - 14\hat{k})$$

$$\because \vec{p}\cdot\vec{c} = 18$$

$$\Rightarrow \lambda(64+1-56) = 18 \Rightarrow \lambda = 2$$

$$\therefore \vec{p} = 2(32\hat{i} - \hat{j} - 14\hat{k}) \quad \text{Ans}$$

⑦ Let $\vec{r_1} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, $\vec{r_2} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

$$\vec{p} = \vec{r_1} + \vec{r_2}$$

$$= (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\hat{p} = \frac{\vec{p}}{|\vec{p}|} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 36 + 4}}$$

$$\because \vec{a}\cdot\hat{p} = 1$$

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k})\cdot\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = 1$$

$$\Rightarrow 2+\lambda+6-2 = \sqrt{(2+\lambda)^2 + 40}$$

$$\Rightarrow (\lambda+6)^2 = (\lambda+2)^2 + 40$$

$$\Rightarrow \cancel{\lambda^2} + 36 + 12\lambda = \cancel{\lambda^2} + 4 + 4\lambda + 40$$

$$\Rightarrow 8\lambda = 8 \Rightarrow \boxed{\lambda = 1} \quad \text{Ans}$$

(8) $\because \vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b} = |\vec{c}|^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta = |\vec{c}|^2$

$\Rightarrow 9+25+2\times3\times5\cos\theta = 49$

$\Rightarrow \cos\theta = 1/2 \Rightarrow \boxed{\theta = 60^\circ}$ Ans

(9) Same Q-7.

$\lambda = 1$

$\therefore$ unit vector along $\vec{b}+\vec{c}$

$$\hat{p} = \frac{(2+1)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+1)^2+36+4}} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{7} \text{ Ans}$$

(10) $\because$ The given vectors are coplanar

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ -1 & \lambda & 2 \end{vmatrix} = 0$$

$\Rightarrow 1(6+\lambda)-1(4-1)+1(2\lambda+3)=0$

$\Rightarrow 3\lambda = -6$

$\Rightarrow \boxed{\lambda = -2}$ Ans

(11) $\because (\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$

$= \vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{d}\times\vec{b}+\vec{d}\times\vec{c}$

$= \vec{c}\times\vec{d}-\vec{b}\times\vec{d}+\vec{b}\times\vec{d}-\vec{c}\times\vec{d}$

$= 0$

$\therefore \vec{a}-\vec{d} \parallel \vec{b}-\vec{c}$

(12) $\because \vec{a}+3\vec{b} \perp 7\vec{a}-5\vec{b}$

$\Rightarrow (\vec{a}+3\vec{b})\cdot(7\vec{a}-5\vec{b})=0$

$\Rightarrow 7\,\vec{a}\cdot\vec{a} - 5\,\vec{a}\cdot\vec{b} + 21\,\vec{a}\cdot\vec{b} - 15\,\vec{b}\cdot\vec{b}=0$

$\Rightarrow 7|\vec{a}|^2 + 16\,\vec{a}\cdot\vec{b} - 15|\vec{b}|^2 = 0$

$\Rightarrow 7|\vec{a}|^2 + 16|\vec{a}||\vec{b}|\cos\theta - 15|\vec{b}|^2 = 0$

$\Rightarrow 7\times1 + 16\times1\times1\cdot\cos\theta - 15\times1=0$

$\Rightarrow -8 + 16\cos\theta = 0$

$\Rightarrow \cos\theta = 1/2$

$\Rightarrow \boxed{\theta = 60^\circ}$ Ans

THREE DIMENSIONAL GEOMETRY (CHAPTER-11)

(1) $\vec{r} = 2\hat{i} - \hat{j} - 2\hat{k}$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{2\hat{i}-\hat{j}-2\hat{k}}{\sqrt{4+1+4}} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}$

$\therefore$ d.c's are $\frac{2}{3}, -\frac{1}{3}, \frac{-2}{3}$ Ans

(2) $\vec{r} = \vec{r_1} + \lambda \vec{m}$

$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + \hat{j} - 2\hat{k})$.

(3) $\cos\theta = \frac{\vec{m}\cdot\vec{n}}{|\vec{m}||\vec{n}|} = \frac{2-1+2}{\sqrt{4+1+1}\cdot\sqrt{1+1+4}} = \frac{3}{6} = \frac{1}{2}$

$\theta = 60°$ Ans

(4) The plane is $\vec{r}\cdot\hat{n} = p$

$\Rightarrow \vec{r}\cdot\hat{i} = 5$ Ans

(5) The line is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$\Rightarrow \frac{x+1}{3+1} = \frac{y-0}{4-0} = \frac{z-2}{6-2}$

$\Rightarrow \frac{x+1}{4} = \frac{y}{4} = \frac{z-2}{4}$

$\therefore$ The line is

$\vec{r} = (-\hat{i} + 2\hat{j}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$ Ans

(6) The lines are

$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2}$ &

$\frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$

The lines are $\perp^r$, $\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow -3\left(-\frac{3p}{7}\right) + \frac{2p}{7}\times 1 + 2\times -5 = 0$

$\Rightarrow \frac{11p}{7} = 10$

$\Rightarrow p = \frac{70}{11}$ Ans

(7) Let the plane is $P_1 + \lambda P_2 = 0$

$$\Rightarrow (3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$$

$$\Rightarrow (3+\lambda)x + (-1+\lambda)y + (2+\lambda)z + (-4-2\lambda) = 0 \quad \text{—(1)}$$

∵ ~~plane (1) is ⊥ to~~ plane (1) is passing through (2, 2, 1)

$$\therefore (3+\lambda)\cdot 2 + (-1+\lambda)\cdot 2 + (2+\lambda)\cdot 1 + (-4-2\lambda) = 0$$

$$\Rightarrow 3\lambda + 2 = 0 \Rightarrow \lambda = -\frac{2}{3}$$

from (1), the plane is

$$3(3x - y + 2z - 4) - 2(x + y + z - 2) = 0$$

$$\Rightarrow \cancel{7x}\ 7x - 5y + 4z - 8 = 0 \quad \text{Ans}$$

(8) Let the line is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ passing through (1, 2, −4)

$$\therefore \frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \quad \text{—(1)}$$

∵ line (1) is ⊥ to the given lines

$$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow 3a - 16b + 7c = 0 \quad \text{—(2)}$$

$$\& \quad 3a + 8b - 5c = 0 \quad \text{—(3)}$$

$$\frac{a}{80-56} = \frac{-b}{-15-21} = \frac{c}{24+48}$$

$$\frac{a}{\cancel{24}\,2} = \frac{b}{\cancel{36}\,3} = \frac{c}{\cancel{72}\,6}$$

∴ from (1), the line is

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \quad \text{Ans}$$

(9) The given line is

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda$$

$$\Rightarrow x = 3\lambda + 2,\ y = 4\lambda - 1,\ z = 2\lambda + 2$$

lie on the plane

$$x - y + z = 5$$

$$\therefore\ 3\lambda + 2 - 4\lambda + 1 + 2\lambda + 2 = 5$$

$$\Rightarrow \lambda + 5 = 5$$

$$\Rightarrow \lambda = 0$$

$\therefore$ pt. of intersection is

$$x = 0+2,\quad y = 0-1,\quad z = 0+2$$

$$= (2, -1, 2)$$

$Q(-1, -5, -10)$

$\cancel{Q(1, 2, -4)}$

$P(2, -1, 2)$

$$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$\cancel{= \sqrt{(2-1)^2 + (-1-2)^2 + (2+4)^2}}$$

$$\cancel{= \sqrt{1+9+36} = \sqrt{46}}$$

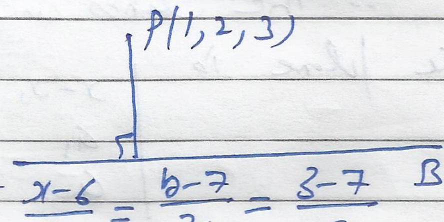
$$= \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2}$$

$$= \sqrt{9+16+144} = \sqrt{169} = 13 \text{ A}$$

---

(10) Line AB is

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$$



$$x = 3\lambda+6,\quad y = 2\lambda+7,\quad z = -2\lambda+7$$

Let pt Q is $(3\lambda+6, 2\lambda+7, -2\lambda+7)$

dr's of AB $= 3, 2, -2$

dr's of PQ $= x_2-x_1,\ y_2-y_1,\ z_2-z_1$

$$= \cancel{3\lambda}\ 3\lambda+6-1,\ 2\lambda+7-2,\ -2\lambda+7-3$$

$$= 3\lambda+5,\ 2\lambda+5,\ -2\lambda+4$$

$\because PQ \perp AB$

$$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow 3(3\lambda+5) + 2(2\lambda+5) - 2(-2\lambda+4) = 0$$

$$\Rightarrow \cancel{9\lambda}\ 17\lambda + 17 = 0$$

$$\Rightarrow \lambda = -1$$

$\therefore$ pt. Q is $(-3+6, -2+7, 2+7)$

$= (3, 5, 9)$

$$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$= \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2} = \sqrt{4+9+36} = \sqrt{49} = 7 \text{ A}$$

(11) The lines are $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$

passing through $(-1,-3,-5)$ & $(2,4,6)$

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} 2+1 & 4+3 & 6+5 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = \begin{vmatrix} 3 & 7 & 11 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= 3(35-28) - 7(21-7) + 11(12-5)$$

$$= 21 - 98 + 77 = 0$$

$\therefore$ The lines are coplanar.

The plane is
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+1 & y+3 & z+5 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0$$

$$\Rightarrow (x+1)(35-28) - (y+3)(21-7) + (z+5)(12-5) = 0$$

$$\Rightarrow 7(x+1) - 14(y+3) + 7(z+5) = 0$$

$$\Rightarrow (x+1) - 2(y+3) + (z+5) = 0$$

$$\Rightarrow x - 2y + z = 0 \text{ A}$$

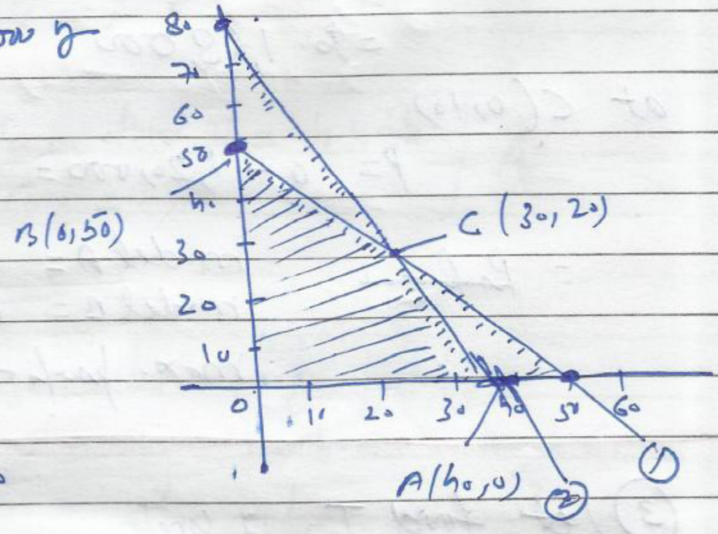
LINEAR PROGRAMMING PROBLEM (CHAPTER-12)

(1) Let Crop X = x, Crop Y = y

$x+y \le 50$ —— (1) ; (0, 50), (50, 0)

$20x+10y \le 800 \Rightarrow 2x+y=80$ —— (2) ; (0, 80), (40, 0)

Max. $P = 10500x + 9000y$



$\because 2x+y=80$

$x+y=50$

$x=30, y=20$

at A(40, 0)

$P = 4,20,000$

at B(30, 20)

$P = 315000 + 180000$

$= 4,95,000$

at C(0, 50)

$P = 0 + 4,50,000$

$\therefore$ for Crop X = 30 hectare, for Crop Y = 20 hectare

max profit = Rs 4,95,000 Ans

---

(2) Let model = x, model B = y

	fabricating	finishing
x	9	1
y	12	3
	≤ 18	≤ 30

~~grinding~~ $9x+12y \le 180$

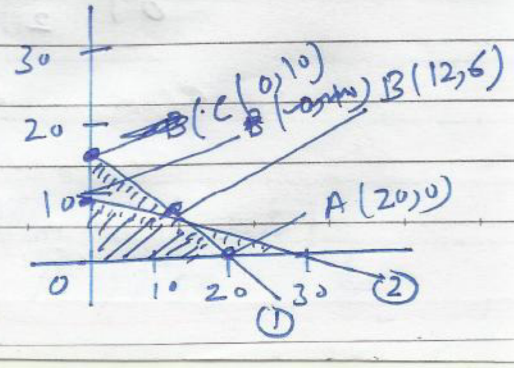
$\Rightarrow 3x+4y \le 60$ —— (1)

~~(0, 3/2) ; (2, 0)~~

& $x+3y \le 30$ —— (2)

$\therefore 3x+4y=60$ $\qquad$ $\therefore x+3y=30$

(0, 15) ; (20, 0) $\qquad$ (0, 10) ; (30, 0)



$\therefore 3x+4y=60$ —— (1)

$3x+9y=90$ —— (2)

$-5y=-30$

$y=6,$ $\qquad$ $3x+24=60$

$3x=36$

$x=12$

Max. (P) = 8000 x + 12000 y

at A(20, 0) ; P = 1,60,000

at B(12, 6)

P = 96000 + 72000

= Rs. 1,68,000

at C(0, 10)

P = 0 + 1,20,000 = Rs 1,20,000

∴ ~~Profit~~ model A = 12 unit

model B = 6 units

max. profit = Rs. 1,68,000 Ans

---

(3) Let food I = x units

food II = y units

		Vit. A	Vit. C
$2x + y \geq 8$ (0, 8), (4, 0)	x	2	1
$x + 2y \geq 10$ (0, 5), (10, 0)	y	1	2
		≥ 8	≥ 10

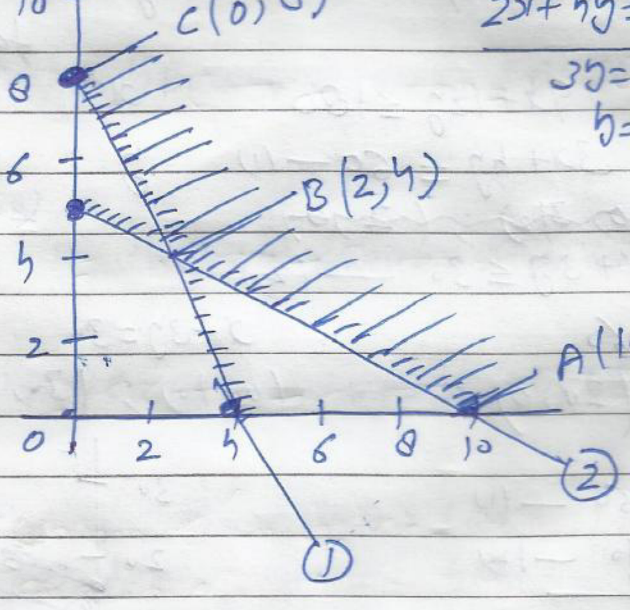
Min (C) = 50x + 70y

$2x + y = 8$ — (1)

$2x + 4y = 20$ — (2)

$3y = 12$

$y = 4, \ x = 2$



at $A(10,0)$

$C = 50\times10+0 = Rs.\ 500$

at $B(2,4)$

$C = 50\times2+70\times4 = 100+280 = Rs\ 380$

at $C(0,8)$

$C = 0+70\times8 = Rs\ 560$

$\therefore$ food I = 2 units, food II = 4 units
& min. Cost = Rs. 380] Ans

(4) Let nuts = x package, bolts = y package.

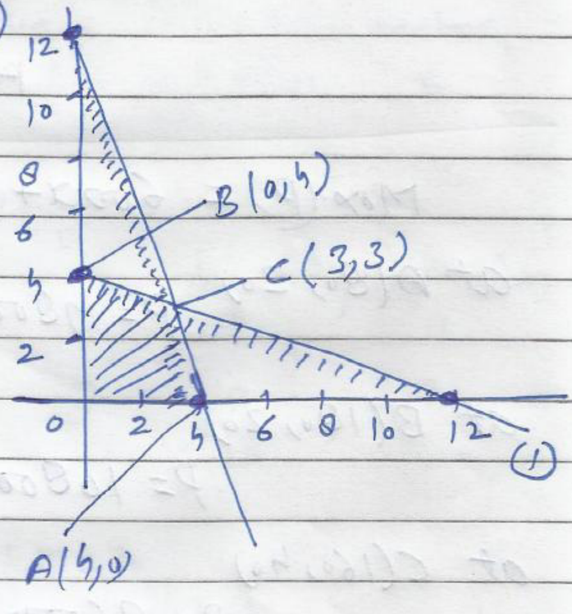
	mach. A	mach. B.
x	1	3
y	3	1
	≤ 12	≤ 12

$x+3y \le 12$ — (1)

$3x+y \le 12$ — (2)

$x+3y=12$; $(0,4)$, $(12,0)$

$3x+y=12$; $(0,12)$, $(4,0)$



$\therefore x+3y=12$

$9x+3y=36$

$8x=24$

$x=3,\ y=3$

Max $(P) = 17.50x+7y$

at $A(4,0)$

$P = Rs\ 70$

at $B(3,3)$

$P = 52.50+21 = Rs.\ 73.50$

at $C(0,4)$

$P = Rs\ 28$

$\therefore$ nuts = 3 packets, bolts = 3 packets
max. profit = Rs 73.50] Ans

(5) Let economy class tickets = x

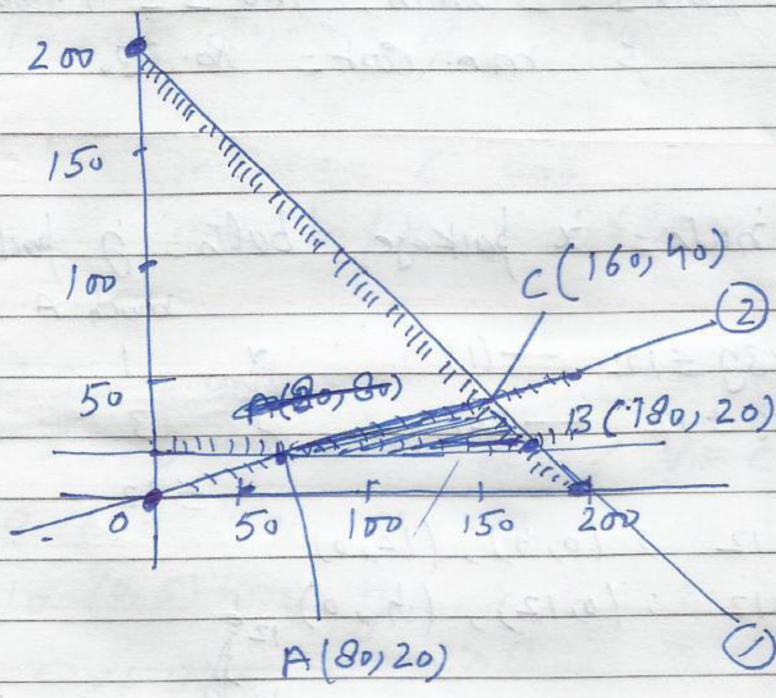
executive " " = y

$x+y \le 200$ —(1) ; (0, 200) ; (200, 0)

$y \geq 20$ —(2)

~~at~~ ~~y~~ $x \geq 4y$

$x-4y \geq 0$ —(3) (0,0) ; (~~80~~, 200, 50)



Max.(P) = $600x + 1000y$

at A(80, 20) P = 48000 + 20000 = Rs. 68000

at B(180, 20)

P = 108000 + 20000 = Rs. 1,28,000

at C(160, 40)

P = 96000 + 40000 = Rs 1,36,000

∴ economy class tickets = 160

executive " " = 40

max. profit = Rs 1,36,000

A₂

CHAPTER-13 (PROBABILITY)

(1) $P(A) = \frac{5}{26}$, $P(B) = \frac{5}{13}$

$\because P(A|B) = \frac{2}{5} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{2}{5}$

$\Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{5+10-4}{26} = \frac{11}{26}$ A

(2) $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$, $P(E \cap F) = \frac{1}{5}$

$P(E) \times P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50} \neq P(E \cap F)$

$\therefore$ E and F are not independent

(3) $\because$ A & B are mutually exclusive $\therefore P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$

$\Rightarrow \frac{3}{5} = \frac{1}{2} + p \Rightarrow p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$ A

(4) $P(A) = 1/4$, $P(B) = 1/2$, $P(A \cap B) = 1/8$

$P(\text{not } A \text{ and not } B) = P(\bar{A} \cap \bar{B})$

$= P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left[\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right] = 1 - \left[\frac{2+4-1}{8}\right] = 1 - \frac{5}{8} = \frac{3}{8}$ A

(5) $P(A) = 1/2$, $P(B) = \frac{1}{3}$

$P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$; $P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(\text{Prob. is solved}) = P(A) \times P(\bar{B}) + P(B) \times P(\bar{A}) + P(A) \times P(B)$

$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{3}$

$= \frac{2+1+1}{6} = \frac{4}{6} = \frac{2}{3}$ A

(6) $S = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (1,H), (1,T), (2,H), (2,T), (4,H), (4,T), (5,H), (5,T), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Let A be the event that at least one die shows a 3.

$A = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$

Let B be the event that the coin shows a head.

$A \cap B = \phi$

$n(A \cap B) = 0$; $n(A) = 6$

$P(B/A) = \dfrac{n(A \cap B)}{n(A)} = 0$ Ans

(7) For a single throw of a pair of dice.

$n(S) = 36$

$E = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(E) = 6$

$p = \frac{6}{36} = \frac{1}{6}$, $q = \frac{5}{6}$, $n = 4$, $r = 2$

$P(X=2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \frac{4 \times 3}{2 \times 1} \times \frac{1}{36_3} \times \frac{25}{36}$

$= \frac{25}{216}$ Ans

(8) $p = \frac{90}{100} = \frac{9}{10}$, $q = 1 - p = \frac{1}{10}$, $n = 10$, $r \le 6$

$P(X \le 6) = \sum_{r=0}^{6} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$ Ans

(9) Kings $= 4$, others $= 48$, Total $= 52$

$r = 0, 1, 2$

$P(X=0) = P(\text{none is King}) = \dfrac{{}^{48}C_2}{{}^{52}C_2} = \dfrac{48 \times 47}{52 \times 51} = \dfrac{188}{221}$

$P(X=1) = P(\text{one is ace and one is other})$

$= \dfrac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \dfrac{4 \times 48 \times 2}{52 \times 51} = \dfrac{32}{221}$

$P(X=2) = P(\text{both are Kings})$

$$= \frac{{}^{4}C_2}{{}^{52}C_2} = \frac{4\times3}{52\times51} = \frac{1}{221}$$

X	$P(X)$	$X\cdot P(X)$	$X^2\cdot P(X)$
0	188/221	0	0
1	32/221	32/221	32/221
2	1/221	2/221	4/221
		34/221	36/221

$\therefore \text{Mean} = \sum X\cdot P(X) = \frac{34}{221} = \frac{2}{13}$

$\text{Variance} = \sum X^2\cdot P(X) - (\text{mean})^2$

$$= \frac{36}{221} - \frac{4}{169}$$

$S.D. = \sqrt{\text{var.}}$

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(10) $P(A) = P(\text{lost card is diamond}) = \frac{13}{52} = 1/4$

$P(B) = P(\text{ " " " not diamond}) = 1 - 1/4 = 3/4$

Let C be the event that two cards drawn from the remaining 51 cards are diamond.

$$P(C/A) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12\times11}{51\times50} = \frac{22}{425}$$

$$P(C/B) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13\times12}{51\times50} = \frac{26}{425}$$

$$P(A/C) = \frac{P(A)\times P(C/A)}{P(A)\times P(C/A) + P(B)\times P(C/B)}$$

$$= \frac{\frac{1}{4}\times\frac{22}{425}}{\frac{1}{4}\times\frac{22}{425} + \frac{3}{4}\times\frac{26}{425}} = \frac{22}{22+78}$$

$$= \frac{22}{100}$$

$$= \frac{11}{50} \text{ Ans}$$

(11) $P(A) = P(\text{ball drawn from bag I is red}) = \frac{3}{7}$

$P(B) = P(\text{" " " " II " black}) = 4/7$

Let C be the prob. that the ball drawn from bag II is red.

$P(C/A) = \frac{5}{10}, \quad P(C/B) = \frac{4}{10}$

$$P(B/C) = \frac{P(B) \times P(C/B)}{P(A) \times P(C/A) + P(B) \times P(C/B)}$$

$$= \frac{\frac{4}{7} \times \frac{4}{10}}{\frac{3}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{4}{10}} = \frac{16}{35+16} = \frac{16}{51} = \underline{A}$$